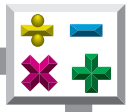


Unit 4: Creating and Interpreting Patterns and Relationships

This unit emphasizes how patterns of change and relationships are used to describe, estimate reasonableness, and summarize information with algebraic expressions or equations to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)

Measurement

- Use a model to derive the formula for circumference. (B.1.3.1)
- Derive and use formulas for finding rate, distance, and time. (B.1.3.2)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships to, geometric shapes in two dimensions. (C.1.3.1)
- Identify and plot ordered pairs in a rectangular coordinate system. (C.3.3.2)

Algebraic Thinking

- Describe a wide variety of patterns and relationships through tables and graphs. (D.1.3.1)
- Create and interpret tables, graphs, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Use algebraic problem-solving strategies to solve real-world problems. (D.2.3.2)

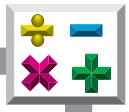
Lesson Purpose

Lesson One Purpose

- Create and interpret tables to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through tables. (D.1.3.1)
- Derive formulas for finding rates, distance, and time. (B.1.3.2)
- Add, subtract, multiply, and divide whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Two Purpose

- Create and interpret graphs to explain cause-and-effect relationships. (D.1.3.2)
- Describe patterns and relationships through graphs. (D.1.3.1)



- Identify and plot ordered pairs in a rectangular coordinate system (graph). (C.3.3.2)
- Use estimation strategies to predict results and to check for reasonableness of results. (A.4.3.1)

Lesson Three Purpose

- Describe a wide variety of patterns. (D.1.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Use concepts about numbers to build number sequences. (A.5.3.1)
- Understand the basic properties of, and relationships pertaining to, geometry shapes in two dimensions. (C.1.3.1)
- Use a model to derive the formula for circumference. (B.1.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Lesson Four Purpose

- Create and interpret tables and verbal descriptions. (D.1.3.2)
- Represent problems with algebraic expressions. (D.2.3.1)
- Use exponential and scientific notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)

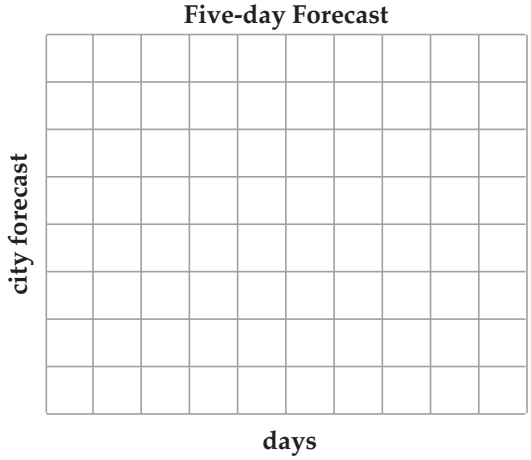
Lesson Five Purpose

- Represent and solve real-world problems graphically and with algebraic expressions and equations. (D.2.3.1)
- Create and interpret tables. (D.1.3.2)
- Use algebraic problem-solving situations to solve real-world problems. (D.2.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use the formulas for finding rate, time, and distance. (B.1.3.2)

Suggestions for Enrichment

1. Have students access the Internet to find information on the past baseball season's American and National leagues' attendance and win statistics. Have students calculate an attendance-to-win ratio (attendance/win rounded to the nearest whole number) for each of the 28 major league teams and determine if winning always leads to good attendance. Then have students plot points for wins on the horizontal axis and attendance on the vertical axis. Do more wins result in greater attendance and why?

2. Have students choose a city and use the Internet to find the five-day forecast of that city's temperature and graph the information.



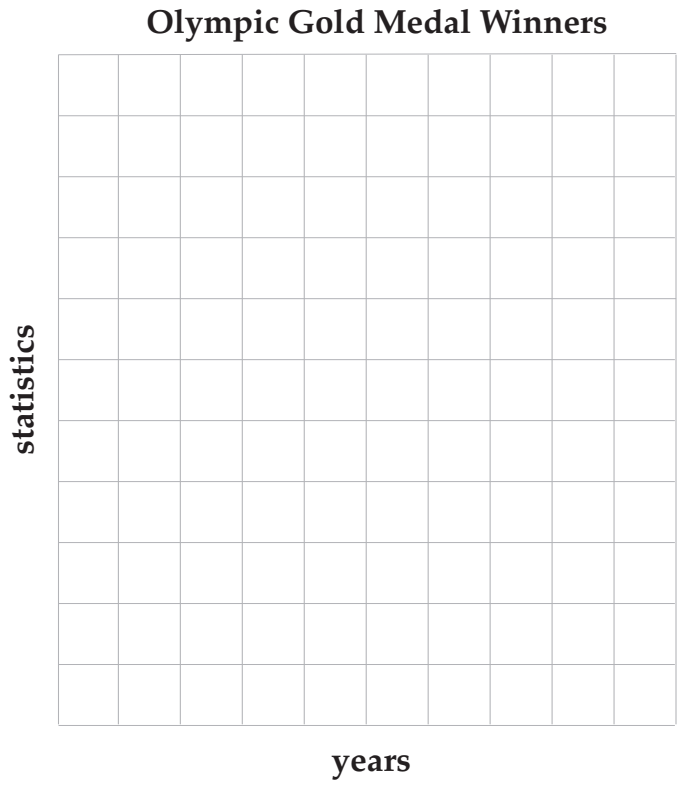
3. Have students access the Internet to find the statistics on all the roller coasters at Six Flags in Georgia. Ask students to calculate which is the highest and fastest roller coaster at Six Flags by computing the average speed rate = distance/time ($r = d/t$) and converting feet/minute to miles/hour. Have students record other interesting facts.

   Have students display data in a summary table as follows.

Six Flags Coaster Computation				
	ride name	ride name	ride name	ride name
height				
length				
distance				
feet/minute				
miles/hour				
extra information				

   Ask students to answer the following: What coaster has the highest drop? What coaster has the highest top speed? What coaster has the slowest average speed? What is the difference between the highest and lowest average speeds? What is the highest average speed of the coasters at Six Flags? What is your favorite coaster? Why (in terms of this activity)?

4. Have students use the Internet to compare records of Olympic gold medalists in one event for the last 100 years or provide them with a set of medalists. Ask students to plot the event on a two-dimensional graph with years on the horizontal axis and statistics on the vertical axis. Then have students answer the following: What trends or patterns did you notice? Were there any years that did not fit the overall picture? Did your trends match other students' trends? Did they have data that did not fit their pattern? Explain your pattern and why you think it happened. Construct an equation that describes the pattern of your data.



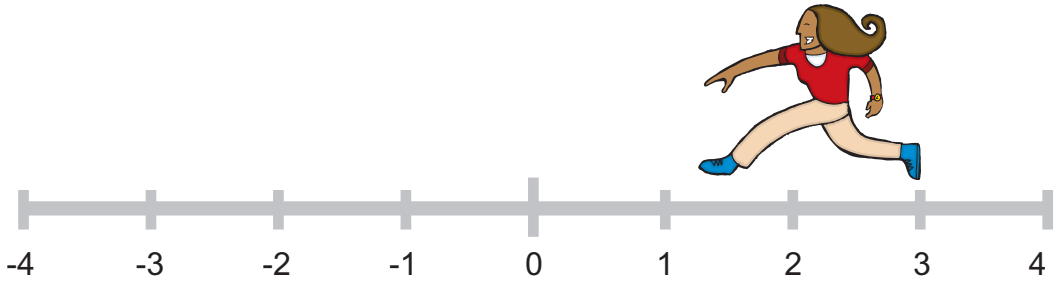
5. Show students a unit cube and ask them to describe the cube (e.g., eight corners, six faces, 12 edges). Have students build a second cube around the first cube so that first cube is encased by the second and then describe it in writing. Ask students how many unit cubes it will take to build a third cube around the second cube, a fourth cube around the third, and so on up to a tenth cube.

To extend the activity: Ask students to imagine that the entire outside of the tenth cube has been painted. If the cube is taken apart into unit cubes, how many faces of cubes are painted on three faces, two faces, one face, no faces? Have students chart their finding for each cube, first through tenth, and look for patterns.

Have students write exponents for the number of cubes needed and painted on three faces, two faces, one face, or no faces. Then have students graph their findings for each dimension of cube, first through tenth, and look for graph patterns.

6. Spray paint lima beans red on one side only. Give each student 10 painted lima beans and a cup for storing and tossing the beans. Have students decide which color will stand for negative and which positive. Working in pairs, one student will toss the beans and the other will record the score. The score is obtained by pairing the white and red beans to make zeros and counting what is left (For example, if the white side is positive, and there is a toss of seven whites and three reds, the three reds pair up with the three whites to make zeros, with four whites left. The score is +4 because +7 + -3 = +4). Have students play at least 15 rounds in order to see some patterns and arrive at a set of rules that can be used with all examples. This will enable them to predict outcomes correctly (e.g., you can find the difference between the units and use the sign of the greater number of units). Discuss. Record on the board several examples written as equations. (Optional: Extend the activity. Subtract, multiply, and divide using lima beans.)

7. Create a number line on the classroom floor using masking tape. Indicate zero (0), the directions of positive (+) and negative (-), and mark integers at intervals of about two feet. Have students take forward steps for positive and backward steps for negative on the line to solve addition of integers. Record movement on number lines on paper.



8. Tell students to imagine that they have been asked to choose between two salary options.

   - One cent on the first day, two cents on the second day, and double their salary every day thereafter for thirty days; or
   - $1,000,000 after 30 days.

   After choosing an option, ask students to complete a table for the first option with columns for day number, pay for the day, and total pay in dollars. (In 30 days this option increases from one penny to over 10 million dollars!)

Option #1 of Pay

Day Number	Pay for That Day	Total Pay (in dollars)
1	.01	.01
2	.02	.03
3	.04	.07
4	.08	.15
etc.	etc.	etc.

9. Ask students which they would choose to receive.

   - $4.50 per day for 30 days; or
   - one penny the first day, two the second day, four the third day, with the amount doubling every day for 30 days.

   Ask the students to compare the two methods on a spreadsheet and graph the results.

10. Pose the following question to students.

    - You have taken a sip from a friend's soda and picked up a bacterium from your friend. If the bacterium divides once every 20 minutes how many potential bacteria could you host in 24 hours? In 48 hours?

    Have students use spreadsheets to explore exponential growth. Have students research growth rates for different species such as bacteria, flies, cats, dogs, or people and transfer the information to a spreadsheet and then graph it. For example, census data can be found in the *World Almanac* or from the United States Bureau of Census (http://www.census.gov). The population data can be used to make projections and then compared to professional projections.

11. See Appendices A, B, and C for other instructional strategies, teaching suggestions, and accommodations/modifications.

Unit Assessment

Circle the letter of the correct answer.

1. Venice and Gloria were jumping rope, and Venice jumped one-third as many times as Gloria jumped. Let j represent the number of jumps Venice made. Which expression could be used to determine the number of jumps for Gloria?

   a. $3 + j$
   b. $j \div 3$
   c. $3j$
   d. $j - 3$

2. In Mr. Faulkner's class, 10 marbles are placed in a jar if every student is seated when the bell rings for class to begin, 5 marbles if every student is in classroom but some are not seated, and no marbles if one or more students are late. When the jar has 500 marbles in it, Mr. Faulkner provides refreshments for his class. The jar now has 420 marbles.

   Tardies have occurred 4 times, and all students have been seated 36 times. Which equation could be used to determine how many days all students were in the classroom on time but some were not seated if n represents this number of days?

   a. $4(0) + 5n = 420$
   b. $4(0) + 36n = 420$
   c. $4(0) + 5n + 36(10) = 420$
   d. $4(0) + 36n + 5(10) = 420$

3. There are 5 pansy plants in each of two rows in the yard of the 1st house on the street.

At the second house there are 7 plants in each two rows, and at the 3rd house, there are 9 in each two rows. If this pattern continues, which house will have 30 plants total?

a. 4th house
b. 5th house
c. 6th house
d. 7th house

4. What is the 6th number in this sequence? 1, 4, 9, 16, _____, _____ .

a. 23
b. 30
c. 32
d. 36

Complete the following.

5. What is the next number in this sequence?

0.25, 0.75, 2.25, 6.75, _____ .

6. The following table shows the amount of pay Leroy earns depending on how many hours he works.

Leroy's Earnings

Number of Hours Worked	Payment of Services
0	0
1	6.25
2	12.50
3	18.75
4	25.00
5	31.25
6	_____
7	_____

Determine his pay for 6 hours. _____

Determine his pay for 7 hours. _____

Explain in words or write an algebraic expression of how his pay for any number of hours could be determined. _____

_____

_____

_____

7. The Jackson family plans a trip of 800 miles and will drive 415 miles the first day. If they average 55 miles per hour on the second day, how long will they travel the second day to reach their destination?

_____

8. Mike's Pizza Place charges $7.00 for a large cheese pizza and $1.00 for each topping. Arlene's Pizza Palace charges $9.00 for a large cheese pizza and $0.75 for each topping. Write an algebraic equation for each company to determine cost for pizza. Let c represent the cost and t represent the number of toppings.

Mike's Pizza Place $c =$ _____

Arlene's Pizza Palace $c =$ _____

Use the two equations to complete the following table.

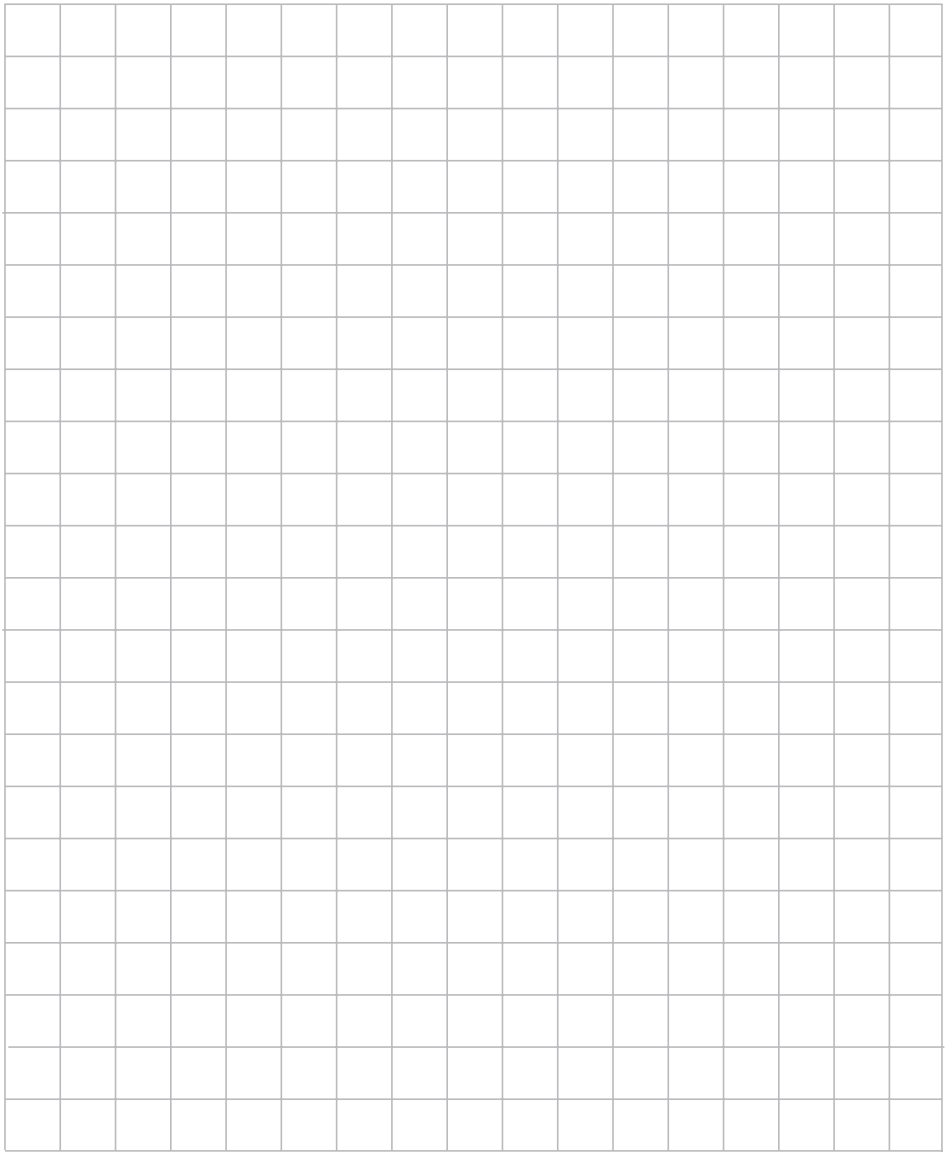
Cost of Pizzas

Number of Toppings	Cost at Mike's Pizza Place	Cost at Arlene's Pizza Palace
0	$7.00	$9.00
1		
2		
3		
4		
5		
6		
7		
8		
9		

9. A person wanting 3 toppings would find _____ (Mike's or Arlene's) a better buy.

10. A person wanting 9 toppings would find _____ (Mike's or Arlene's) a better buy.

11. Make a graph of the data and circle the point on the graph where the charge is the same at both companies. Be sure to title your graph, label your axes, and include a key.



12. Attach to this test the task assigned at the beginning of the unit on page 264. Be sure that the pattern you found interesting is described, a table and graph are provided, and conclusions and projections are written.

Keys

Lesson One

Practice (pp. 266-268)

1. True
2. True
3. True
4. True
5. True
6. True
7. True
8. False; all entries are not divisible by 100
9. True
10. True
11. True
12. True
13. False; the distance traveled in 18 hours would be 900 miles
14. True
15. True

Practice (p. 269)

See table below.

Time in Hours	Distance Traveled at 50 mph	Distance Traveled at 65 mph
0	0	0
1	50	65
2	100	130
3	150	195
4	200	260
5	250	325
6	300	390
7	350	455
8	400	520
9	450	585
10	500	650
11	550	715
12	600	780
13	650	845*
14	700	910*
15	750	975*

*Students may not have these entries since original problem was based on maximum distance of 750 miles.

Practice (p. 270)

Answers will vary.

Practice (pp. 271)

1. 9.7 to 10; 7.5
2. 13.5; 10 to 10.4
3. 5.5 to 6; 4 to 4.5

Practice (p. 273)

1. Yes; the conjecture should be found true with perhaps statements and work shown to illustrate
2. Yes; formulas for distance, time and rate should be written in words and symbols as directed

Practice (pp. 274-275)

1. Answers will vary but may include the following: The table would be helpful if you were using it to answer many questions and could simply refer to the table for your answers.
2. Answers will vary but may include the following: Dividing the distance by the rate to determine the travel time would be helpful if you were finding travel time for one or maybe a few questions.
3. Decreasing speed increases travel time.
4. Answers will vary.
5. Answers will vary.

Practice (p. 276)

1. B
2. D
3. E
4. C

Keys

5. F
6. A

Practice (p. 277)

1. B
2. A
3. C
4. E
5. D

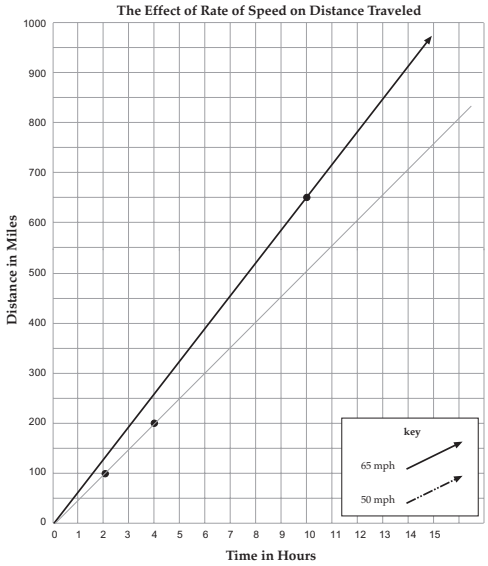
Lesson Two

Practice (pp. 282-283)

1. True
2. True
3. True
4. True
5. True
6. True
7. False; the distance for 7 hours at 65 mph is 455 or about 450, not 550
8. False
9. True
10. False; a key is necessary
11. True
12. True

Practice (p. 284)

See graph below. Fifty miles per hour key symbols will vary.



Practice (pp. 285-286)

1. (0; 0); 0 hours; 0 miles
2. more
3. above
4. 50 mph; 65 mph
5. increases
6. increases
7. 700 miles to 750 miles
8. 5.5 hours
9. somewhat like
10. 150; somewhat like

Keys

Practice (p. 287)

1. Answers will vary but may include the following: If I want specific information, I might use the table. An example might be to find distances traveled at 55 mph at 3, 4, 5, 6, and 7 hours.
2. Answers will vary but may include the following: If I want the "big picture" of the overall effect of rate on distance traveled, the graph is helpful. I can also secure specific information from the graph also but they might be more convenient with the table.
3. Answers will vary but may include the following: Estimation was used when plotting a point that did not lie at an intersection on the grid. It was also used when determining distance traveled for a given time or time for a given distance using the graph in Lesson Two.

Practice (p. 288)

1. G
2. E
3. F
4. A
5. B
6. C
7. D

Lesson Three

Practice (pp. 290-294)

1. 49
2. 29, 31, 37
3. 4, 2, 1
4. 56, 92; square
5. 10 15

X X

X X X X

X X X X X

X X

X X X

X X X X

X X X X X

6. 243, 729; Multiply an entry by 3. (All entries are powers of 3.)
7. ⅞, Ⅷ or 1; Add ⅛ to entry.
8. 1.75, 2.10; Add 0.35 to an entry to get the next one or multiply 0.35 by 1 for the first entry, by 2 for the second entry, etc.
9. 7½, 8 ¾; Multiply 1¼ by 1 for the first entry, 1¼ by 2 for the second entry, etc. Each entry is 1¼ more than the previous one. Or add 1¼ to entry.
10. 12345654321
11. Let students share their five favorites from page 289 and discuss.

Practice (p. 295)

1. C
2. D
3. H
4. A
5. F
6. B
7. E
8. G

Practice (pp. 296-304)

1. 900; 1,080; The sum of interior angle measures is 180 times the difference in the number of sides and 2. (Students may say to add 180 to the previous entry. Help guide thinking to enable response for 10-sided polygon without building table.)
2. 15.7; 18.84; The circumference of a circle is the diameter multiplied by 3.14. (Students may say to add 3.14 to the previous entry. Guidance is needed as in #1.)
3. 16; 1+2+4+8=15;
   32; 1+2+4+8+16=31;
   64; 1+2+4+8=+16+32= 63;
   For powers of two, the sum of the proper factors is one less than the

value of the exponential expression. (Students may say to add value of previous power of two to previous sum of proper factors. Guidance is needed as in #1.)

4. 3.5 square units; 4 square units; The area of the triangle is one-half the area of the "related parallelogram."
5. 10 cubic units; 12 cubic units; 2/3
6. 28 square units; 32 square units; 4; Sketches will vary.
7. 63 square units; 72 square units; 9; Sketches will vary.
8. 56 cubic units; 64 cubic units; 8
9. Answers will vary.
10. Answers will vary.

Practice (p. 305)

1. L
2. A
3. E
4. I
5. D
6. J
7. H
8. K
9. F
10. G
11. B
12. C

Lesson Four

Practice (pp. 307-312)

1. 3; 5; 7; 9; 11
2. 1; 3; 5; 7; 9; $2n - 1$
3. 1; 16; 81; 256; 625
4. 3; 6; 9; 12; 15; $3n$
5. 95; 195; 295; 395; 495
6. ¼ or .25; ¾ or 0.5; ¾ or 0.75; 1; 5/4 or 1 ¼ or 1.25; $n/4$
7. 1/10 or 0.1; 2/10 or 0.2; 3/10 or 0.3; 4/10 or 0.4; 5/10 or 0.5
8. 10; 17; 24; 31; 38; $7n + 3$
9. 28; 35; 42; 49; 56
10. 1; 2; 3; 4; 5; $(n/4)4$

Practice (p. 313)

Answers will vary.

Practice (pp. 315-319)

Answers may vary. Students may provide expressions other than those provided here. If so, check their responses for validity.

1. n+5
2. n-1
3. $4n$
4. n^2
5. $3n$+1
6. $2n$-2
7. $10n$
8. $n/2$ or $\frac{1}{2}n$
9. n^3

Practice (p. 320)

Answers will vary.

Practice (pp. 321-322)

1. .15; 15%
2. $556.92
3. $1,092.00
4. $7,280.00
5. $5,361.08
6. 22.65%

Lesson Five

Practice (pp. 325-328)

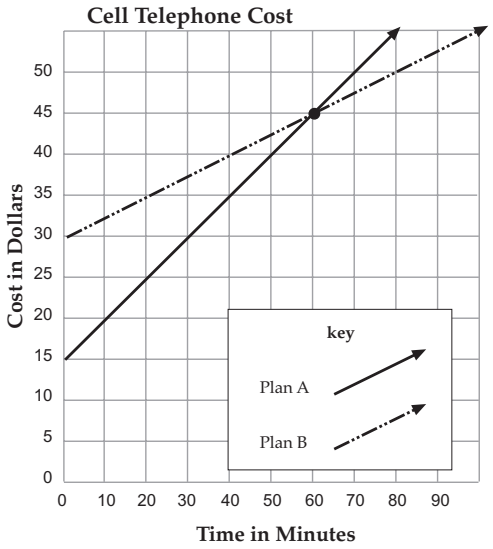
1. Plan A: $15 + 0.50m$
   Plan B: $30 + 0.25m$
2. He should use Plan A. It would cost $20 for 10 minutes while Plan B would cost $32.50.
3. He should use Plan B. The cost for Plan A is $45.50 for 61 minutes, while for Plan B it is $45.25. Another example is that at 70 minutes the cost for Plan A is $50

and for Plan B it is $47.50.

4. See table below.

Minutes of Usage	Plan A	Plan B
0	15	30
10	20	32.50
20	25	35
30	30	37.50
40	35	40
50	40	42.50
60	45	45
70	50	47.50

5. See graph below.



6. *m* (which stands for minutes) can be replaced in the rule (for either plan) with the number of minutes, and cost can be determined. Answers will vary but may include the following: The organization of the data in the table allows quick comparisons of cost; The graph provides a visual representation with one line representing the better plan for 0 to 60 minutes, the other line for more than 60 minutes, while the intersection of the two lines highlights the point at which both plans have the same cost; Answers will vary.

Practice (p. 329)

1. 1368 miles per day
2. 57 mph

Unit Assessment (pp. 95-99TG)

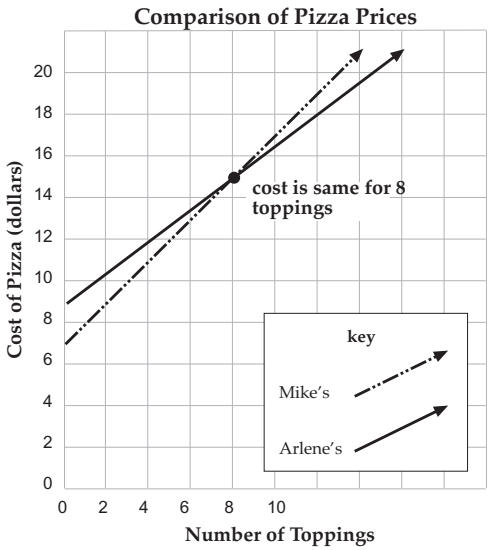
1. c
2. c
3. c
4. d
5. 20.25
6. $37.50; $43.75; The hourly rate of $6.25 can be multiplied by the number of hours worked or 6.25*n* where *n* represents the number of hours worked.
7. 7 hours
8. 7 + 1*t*; 9 + .75*t*; See table below.

Cost of Pizza

Number of Toppings	Mike's Pizza Place	Arlene's Pizza Palace
0	$7.00	$9.00
1	$8.00	$9.75
2	$9.00	$10.50
3	$10.00	$11.25
4	$11.00	$12.00
5	$12.00	$12.75
6	$13.00	$13.50
7	$14.00	$14.25
8	$15.00	$15.00
9	$16.00	$15.75

Keys

9. Mike's
10. Arlene's
11. See graph below. Key symbols will vary.



12. Answers will vary.

Scoring Recommendations for Unit Assessment

Item Numbers	Assigned Points
1	6 points
2	7 points
3	6 points
4	7 points
5	7 points
6	6 points for table; 6 points for explanation
7	7 points
8	6 points for algebraic expressions
9	6 points for table (if errors are present in table, credit may be given for analysis and graph if based on information in table)
10	6 points for analysis
11	6 points for graph
12	6 points for pattern; 6 points for table; 6 points for graph; 6 points for conclusions/conjectures
	Total = 100 points

Benchmark Correlations for Unit Assessment

Benchmark	Addressed in Items
D.1.3.1	3, 4, 5, 6
D.1.3.2	1, 2
D.2.3.1	8, 9, 10, 11, 12
D.2.3.2	7, 8, 9, 10, 11
A.3.3.3	7, 8
A.5.3.1	4, 5, 6
B.1.3.2	6, 7, 8
C.3.3.2	11