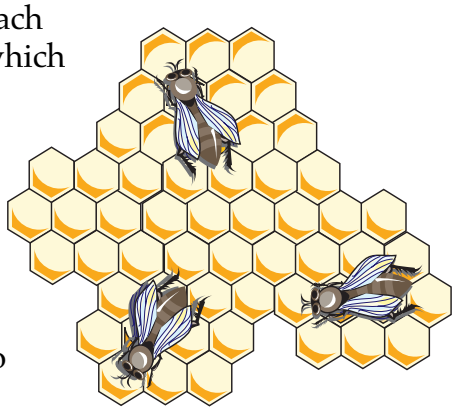
Tessellations

Look at the honey bees in their beehive. Each hexagon shape in the beehive is a cell in which the bees store honey. This pattern of hexagons is an example of a **tessellation**.



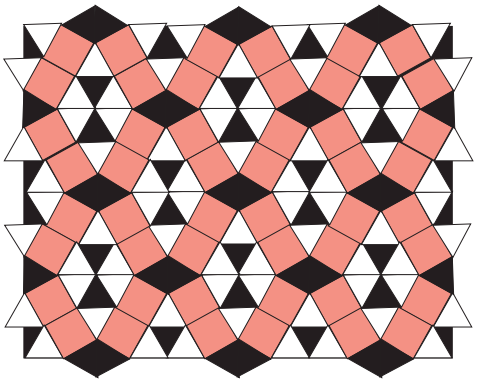
A *tessellation* is a covering or tiling of a **plane** (a two-dimensional surface) by congruent copies of the same shape. The shape must be repeated and fit together so that there are no holes and no overlaps between shapes. Tessellations can be formed by combining **translations** (**slides**), **rotations** (**turns**), and **reflections** (**flips**) of images.

In the beehive above, each cell is a *regular hexagon*. A *regular polygon* is a polygon with *all sides the same length* and *all angles the same measure*. It is both *equilateral* and *equiangular*. A regular polygon with six sides is a regular hexagon.

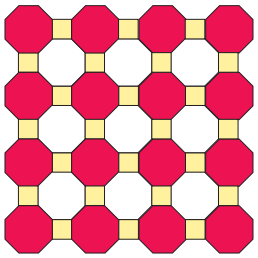
A *regular tessellation* uses congruent regular polygons of only one kind. There are three regular polygons that tessellate: squares, equilateral triangles, and hexagons.

All the vertices of the figures fit right next to each other around a point. The sums of the angles around any one point equal 360 degrees.

A *semiregular tessellation* uses congruent regular polygons of more than one kind. There are eight semiregular tessellations, two of which use mirror images of each other.



triangle and squares



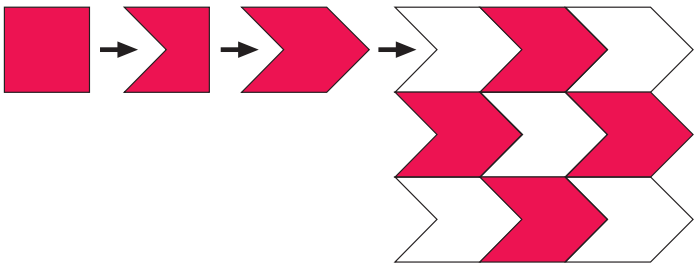
octagons and squares

examples

Variations of these regular polygons can also tessellate. If you change one side and then change the opposite side to match in the same way, the shape will tessellate.

Example: Change a square to make a new tessellation.

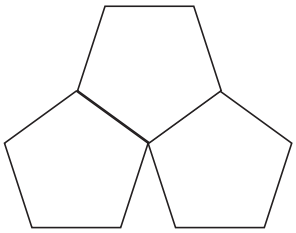
- start with a square
- change one side
- copy the change to the opposite side



Other shapes can be used in tessellations. The Dutch artist Maurits Cornelis Escher (1898-1972) became famous for the unusual shapes he used in tessellations.

There are some shapes that do not tessellate.

Example: congruent regular pentagons



All Answer Keys are included in the Teacher Edition

Practice

Answer the following.

Equilateral triangles will tessellate a plane. All their vertices fit right next to each other around a point. Each angle measures 60 degrees, and 60 is a **factor** of 360. (It divides 360 evenly.) It takes six equilateral triangles to surround a point, and 60 x 6 = 360.

equilateral triangles

1. Name the other two regular polygons that will tile or tessellate the plane. Tell how many of the polygons will be required to surround a point.

   a. _____ will tile a plane because

   _____ is a *factor* of 360. It will take

   _____ of these _____

   to surround a point.

   b. _____ will tile a plane because

   _____ is a factor of 360. It will take

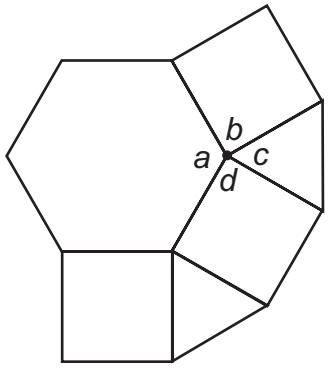
   _____ of these _____

   to surround a point.

2. For each of the patterns for tessellations provided, indicate the measure of each of the angles and find the sum of the angles to verify that it is 360 degrees each time.

a. $a =$ _____ ; $b =$ _____ ; $c =$ _____ ; $d =$ _____

_____ + _____ + _____ + _____ = _____

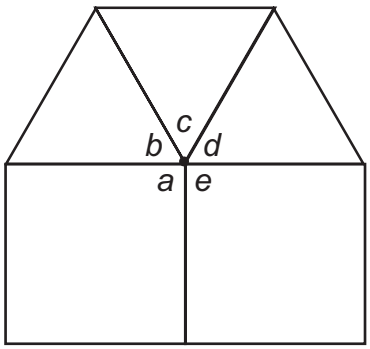


b. $a =$ _____ ; $b =$ _____ ; $c =$ _____ ; $d =$ _____ ;

$e =$ _____

_____ + _____ + _____ + _____ + _____

= _____

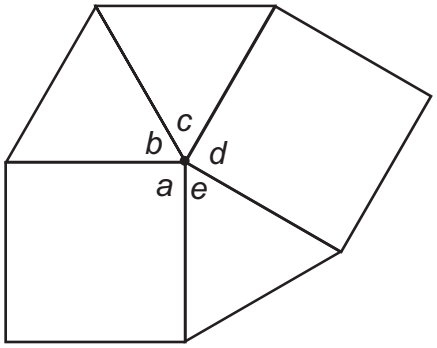


c. $a =$ _____ ; b _____ ; $c =$ _____ ; $d =$ _____ ;

$e =$ _____

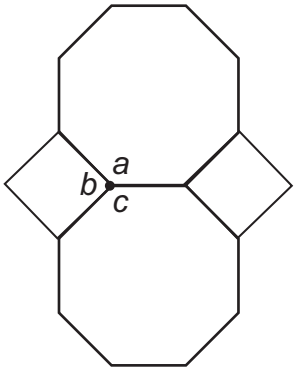
_____ + _____ + _____ + _____ + _____

= _____



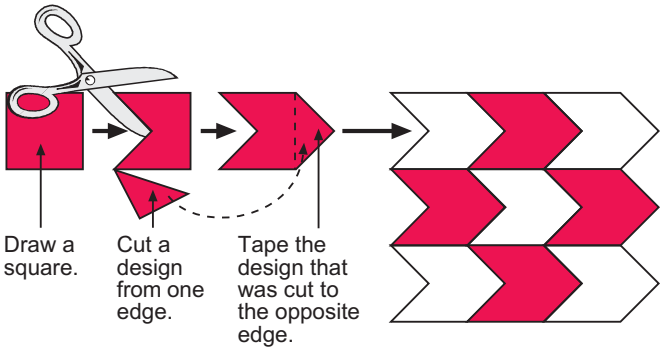
d. a = _____ ; b = _____ ; c = _____

_____ + _____ + _____ = _____



Practice

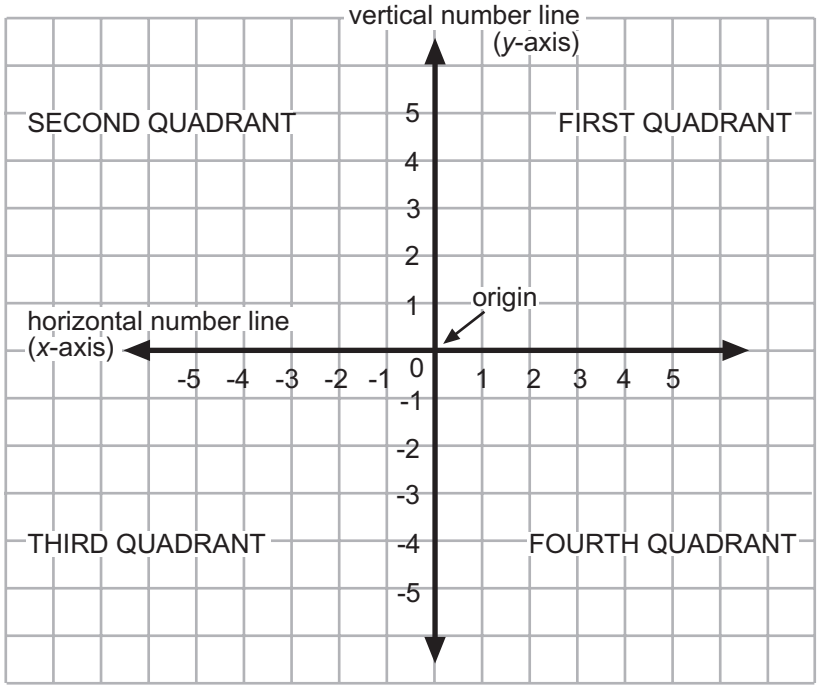
Draw a square on a piece of paper. From one edge of the square, cut an **interesting design** *(see example below). Tape the design that was cut to the* **opposite edge** *of the square. Your new figure should* **tessellate the plane**. *Do the same with a* **parallelogram**.



example

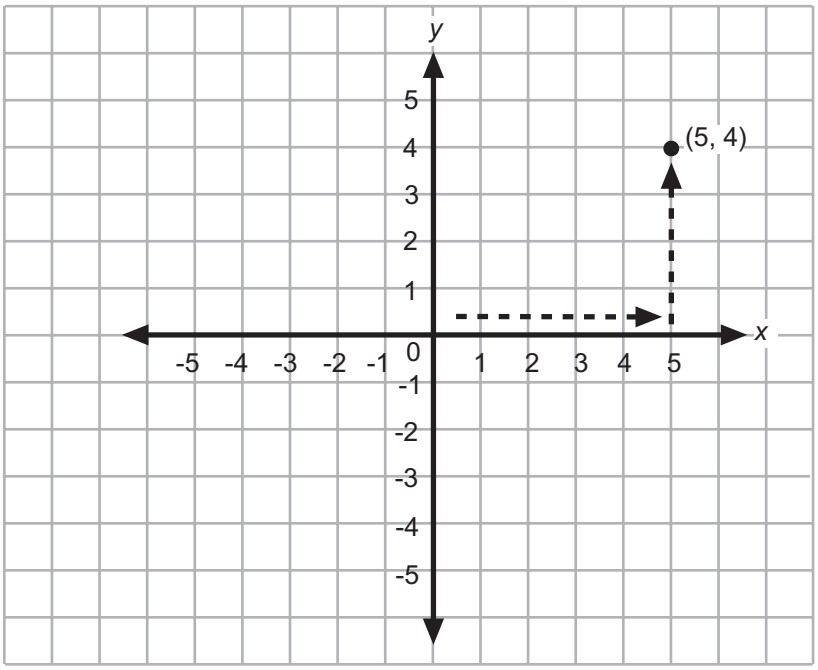
Coordinate Grid or System

A **coordinate grid or system** is a two-dimensional system used to locate points in a plane. A *coordinate grid or system* has a horizontal (↔) **number line** (***x*-axis**) and a vertical (↕) number line (***y*-axis**). These two number lines or **axes of a graph** *intersect* or meet at the **origin**. The **coordinates** at the **intersection** of the origin are (0, 0.) The axes form four regions or **quadrants**. However, the origin and the axes are not in any quadrant.



To locate **ordered pairs** or coordinates such as (5, 4) on a coordinate system, do the following.

- start at the origin (0, 0) of the **grid**
- locate the first number of the ordered pair or the ***x*-coordinate** on the *x*-axis (⟷)
- then move *parallel* (||) to the *y*-axis and locate the second number of the ordered pair or the ***y*-coordinate** on the *y*-axis (↕) and draw a *point*



Practice

Use the list below to write the correct term for each definition on the line provided.

axes (of a graph)	**number line**	***x*-axis**
coordinates	**ordered pair**	***x*-coordinate**
coordinate grid	**origin**	***y*-axis**
intersection	**quadrant**	***y*-coordinate**

_____ 1. the first number of an ordered pair

_____ 2. the second number of an ordered pair

_____ 3. any of four regions formed by the axes in a rectangular coordinate system

_____ 4. the point at which two lines meet

_____ 5. numbers that correspond to points on a graph in the form (x, y)

_____ 6. the intersection of the x-axis and y-axis in a coordinate plane, described by the ordered pair (0,0)

_____ 7. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

_____ 8. the vertical ($\updownarrow$) axis on a coordinate plane

_____ 9. the horizontal ($\longleftrightarrow$) axis on a coordinate plane

_____ 10. a line on which numbers can be written or visualized

_____ 11. the location of a single point on a rectangular coordinate system where the digits represent the position relative to the x-axis and y-axis
Example: (x, y) or (3, 4)

_____ 12. network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps