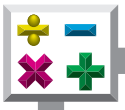
Lesson Two Purpose

- Associate verbal names, written word names, and standard numerals with integers, rational numbers, irrational numbers, and real numbers. (MA.A.1.4.1)
- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including square roots, exponents, and appropriate inverse relationships. (MA.A.3.4.1)
- Add, subtract, multiply, and divide real numbers, including exponents, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (MA.A.3.4.3)

Variables and Expressions

Suppose you are n years old today. In 4 years, your age can be described by the expression $n + 4$. Two years ago, your age would have been $n - 2$.

The letter n is a **variable**. A *variable* is any symbol that could represent a number. In this example, the variable represents your current age. Note that *any* letter of the alphabet or symbol can be used as a variable. A combination of operations, variables, and numbers is called a mathematical expression, algebraic expression, or simply an *expression*.



Here are sample phrases used to write mathematical expressions.

	Word Expression	**Mathematical Expression**
Addition:	5 **increased** by a number n	$5 + n$
	a number y plus 2	$y + 2$
	a number t increased by 4	$t + 4$
	the **sum** of a number b and 5	$b + 5$
	10 more than a number m	$m + 10$
Subtraction:	a number x minus 2	$x - 2$
	a number n less 3	$n - 3$
	5 less than a number t	$t - 5$
	a number t less than 5	$5 - t$
	a number c **decreased** by 2	$c - 2$
	the **difference** of a number x and 5	$x - 5$

	Word Expression	Mathematical Expression
Multiplication:	4 times a number y (form used most often is $4y$)	$4 \times y$, $4(y)$, $4 \bullet y$, or $4y$
	the product of 3 and a number n	$3n$
	6 multiplied by a number t	$6t$
	twice a number p	$2p$
	$\frac{1}{2}$ a number y	$\frac{1}{2}y$
Division:	a number y divided by 2	$\frac{y}{2}$
	the *quotient* of t and 4	$\frac{t}{4}$
	a number c divided by 3	$\frac{c}{3}$
	3 divided by a number c	$\frac{3}{c}$
Power:	the square of x	x^2
	the cube of a	a^3
	the fourth power of x	x^4

Remember: $5n = 5 \times n$, $5(n)$, $5 \bullet n$

$\frac{x}{3} = x \div 3$

Practice

Write a **mathematical expression** *for each* **word expression**.

1. 8 increased by a number y _____
2. 7 less than a number d _____
3. 15 decreased by s _____
4. 5 more than a number t _____
5. the sum of a number y and 4 _____
6. 12 less a number x _____
7. the product of 8 and a number d _____
8. 30 divided by a number b _____
9. the sum of a number r and 10 _____
10. a number t minus 6 _____
11. the quotient of 8 and a number c _____
12. 10 times a number y _____
13. twice a number q _____
14. the square of b _____
15. the cube of p _____

Read the following.

Study These Expressions	
Words	**Symbols**
three times x plus y	$3x + y$
three times the sum of x and y	$3(x + y)$

In word expressions, look for key words that indicate that parentheses ( ) are to be used. Sometimes the words *sum*, *difference*, *quantity*, and *total* signal the use of parentheses.

Write a **mathematical expression** *for each* **word expression**. *Use* **parentheses ( )** *where appropriate.*

16. twice the sum of a number and 7 _____

17. one-half of the difference of a number x and 10 _____

18. twice a number increased by 8 _____

19. twice the total of a number and 5 _____

Answer the following.

20. One of the following does *not* belong. Write a sentence explaining why.

    a. Multiply 5 and a number then subtract 7.
    b. Subtract the product of 5 and a number from 7.
    c. $5x - 7$
    d. 7 less than the product of 5 and a number.

_____

_____

_____

Evaluating Expressions

Here is how to evaluate mathematical expressions.

Suppose you are 16, and we let your age be represented by the variable a. The variable a now has a given **value** of 16. Calculate your age as follows:

a. in 4 more years — $a + 4 = \mathbf{16} + 4 = 20$

b. divided by 2 — $\frac{a}{2} = \frac{\mathbf{16}}{2} = 8$

c. twice your age increased by 2 — $2a + 2 = 2(\mathbf{16}) + 2 = 32 + 2 = 34$

d. the product of your age and 3 — $3a = 3(\mathbf{16}) = 48$

Suppose you are 16, and we let your age be represented by the variable a.

Practice

Use the given **value** *of each* **variable** *below to evaluate each expression.*

$x = 6$	$y = 8$	$z = 2$

1. $x + 12 =$
2. $y - 5 =$
3. $x + z =$
4. $x - z =$
5. $y - x =$
6. $10 - y =$
7. $x + y - 2 =$
8. $20 - y - z =$
9. $x + x + x =$
10. $10 + x - x =$

Practice

Use the given **value** *of each* **variable** *below to evaluate each expression.*

$x = 6$	$y = 8$	$z = 2$

1. $5y =$

2. $\frac{y}{2} =$

3. $xz =$

4. $\frac{y}{z} =$

5. $5xy =$

6. $\frac{8}{y} =$

7. $5y + 10 =$

8. $\frac{20}{z} - x =$

9. $yz - x =$

10. $\frac{12}{x} + 12 =$

Practice

Use the given **value** *of each* **variable** *below to evaluate each expression.*

$r = 4$	$s = 5$	$t = 10$

1. r increased by s
2. the sum of r and t
3. s less than t
4. t minus r
5. r more than 4
6. t divided by s
7. the product of r and s
8. s decreased by r
9. the sum of s and t decreased by 9
10. 12 divided by r, plus s
11. the cube of s increased by the sum of r and t
12. the square of s decreased by the quotient of t and s

Practice

Use the list below to write the correct term for each definition on the line provided.

decrease	**increase**	**variable**
difference	**sum**	

_____ 1. any symbol that could represent a number

_____ 2. the result of an addition

_____ 3. to make greater

_____ 4. the result of a subtraction

_____ 5. to make less

Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. the result of a multiplication

_____ 2. the result of a division

_____ 3. a collection of numbers, symbols, and/or operation signs that stands for a number

_____ 4. an exponent; the number that tells how many times a number is used as a factor

_____ 5. the result when a number is multiplied by itself or used as a factor twice

_____ 6. the third power of a number

_____ 7. any of the numbers represented by the variable

A. cube

B. expression

C. power (of a number)

D. product

E. quotient

F. square (of a number)

G. value

Lesson Three Purpose

- Understand and explain the effects of addition, subtraction, multiplication, and division on real numbers, including exponents and appropriate inverse relationships. (MA.A.3.4.1)
- Select and justify alternative strategies, such as using properties of numbers, including inverse, identity, and associative, that allow operational shortcuts for computational procedures in real-world or mathematical problems. (MA.A.3.4.2)

Solving Equations by Guessing

An **equation** is a mathematical sentence that *equates* one expression to another expression.

For example, you know:

$2 + 2 = 4$
$2 \bullet 3^2 = 18$

Now, consider this *equation*:

$2(x + 3) = 14$

What number could I use in place of the variable x, so that the left side is equal to the right side? We can guess. It must be a number that when multiplied by 2 equals 14.

2 x ? = 14

We know that

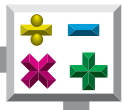
$2 \bullet 7 = 14$,

and we know that

$(4 + 3) = 7$.

Therefore "4" is a **solution** to this equation.

4 is the *value* of the variable x.
$x = 4$



The equation is **solved** by **substituting** or *replacing x* in the original equation with the value of 4.

$$\begin{aligned} 2(x + 3) &= 14 \\ 2(4 + 3) &= 14 \\ 2(7) &= 14 \\ 14 &= 14 \end{aligned}$$

The *solution* of 4 makes the equation true.

Finding the value of a variable that makes a mathematical sentence true is called *solving the equation*. The value of the variable is called *the solution of the equation*.

Practice

Guess the answers *to the following.*

1. $x + 4 = 20$ (*Think*: What number can you add to 4 to get 20?)
   $x =$

2. $x - 4 = 20$ (*Think*: What number can you subtract 4 from to get 20?)
   $x =$

3. $4x = 20$ (*Think*: 4 times what number is 20?)
   $x =$

4. $\frac{x}{4} = 20$ (*Think*: What number can you divide by 4 to get 20?)
   $x =$

5. $6(x + 3) = 48$
   $x =$

6. $6x^2 = 24$
   $x =$

7. $(2x)^2 = 36$
   $x =$

Decide whether **5 is a solution** *to the following problems. Write* **yes** *if the solution is 5. Write* **no** *if the solution is* not *5.*

_____ 8. $23 - x = 6$

_____ 9. $2x + 3x = 25$

_____ 10. $\frac{130}{x} = 26$

Properties

Guessing is an acceptable way to solve simple equations, but we need to develop strategies which will help us solve harder equations. Before we do this, we need to examine some basic *properties* which will help us work with variables. These properties will help us make the leap from simple to more complex equations.

Order (Commutative Property)	
Commutative Property of Addition: Numbers can be added in any order and the sum will be the same. $10 + 2 = 2 + 10$ $x + 2 = 2 + x$	**Commutative Property of Multiplication:** Numbers can be multiplied in any order and the product will be the same. $2 \cdot 10 = 10 \cdot 2$ $2 \cdot x = x \cdot 2$
Grouping (Associative Property)	
Associative Property of Addition: Numbers can be grouped in any order and the sum will be the same. $(5 + 3) + 2 = 5 + (3 + 2)$ $(5 + x) + y = 5 + (x + y)$	**Associative Property of Multiplication:** Numbers can be grouped in any order and the product will be the same. $(5 \cdot 3) \cdot 2 = 5 \cdot (3 \cdot 2)$ $(5 \cdot x) \cdot y = 5 \cdot (x \cdot y)$
Identity Properties	
Additive Identity: The sum of any number and zero is the number. $5 + 0 = 5$ $x + 0 = x$	**Multiplicative Identity:** The product of any number and one is the number. $5 \cdot 1 = 5$ $x \cdot 1 = x$
Inverse Properties	
Additive Inverse: The sum of any number and its additive inverse is 0. $3 + -3 = 0$ 3 and -3 are additive inverses, also called **opposites**.	**Multiplicative Inverse:** The product of any number and its multiplicative inverse (**reciprocal**) is 1. $4 \times \frac{1}{4} = 1$ 4 and $\frac{1}{4}$ are multiplicative inverses, also called *reciprocals*.

Practice

Answer the following.

1. Why are division and subtraction not listed as commutative operations? Give examples to show your reasoning.

   Answer: _____

   _____

   Examples:

2. Unfortunately, little Ben has lost his calculator and needs to add 4 + 5 + 16 + 15. Since we can add in any order, what would be a quick way to group the numbers and get the sum?

   Answer: _____

   _____

3. Do the following calculation mentally by using the properties on the previous page, then explain your strategy.

   25 • 16 • 4

   Answer: _____

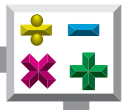
   _____

   Explain: _____

   _____

   _____

   _____



Study the following examples on **simplifying** *before attempting the problems that follow. To* **simplify an expression**, *perform as many of the indicated operations as possible. To* **simplify a fraction**, *write the* **fraction** *in lowest terms or* **simplest form**.

Example one

$(y + 2) + 3 =$	
$y + (2 + 3) =$	associative property of addition
$y + 5$	

Example two

$(7 \bullet x) \bullet 3 =$	
$(x \bullet 7) \bullet 3 =$	commutative property of multiplication
$x \bullet (7 \bullet 3) =$	associative property of multiplication
$x \bullet 21 =$	
$21 \bullet x =$	commutative property of multiplication

Simplify each expression *below using the* **properties** *from page 51.*

4. $(x + 3) + 4 =$

5. $(x \bullet 6) \bullet 5 =$

6. $5 \bullet (4x) =$

7. $11 + (2 + x) =$

8. $(m \bullet 4) \bullet 7 =$

9. $(1 \bullet x) \bullet 5 =$

Answer the following.

10. Is $0 + xy$ the same as yx? _____

    Explain: _____

    _____

    _____

    _____

11. Is $b + 9a$ the same as $9a + b$? _____

    Explain: _____

    _____

    _____

    _____

12. Is $2x - 5y$ the same as $5y - 2x$? _____

    Explain: _____

    _____

    _____

    _____

13. Is $15 \div 3$ the same as $3 \div 15$? _____

    Explain: _____

    _____

    _____

    _____